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On Non-Abelian Thomas-Fermi Screening

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Abstract

The Thomas-Fermi screening of non-Abelian gauge fields by fermions or screening of gluon fields in quark matter is discussed. It is described by an effective mass term which is, as with hard thermal loops, related to the eikonal for a Chern-Simons theory and the Wess-Zumino-Witten action.

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The screening of electromagnetic interactions in a plasma of charged particles, the so-called Debye screening, is quite well-known. An analogous effect, the Thomas-Fermi screening, occurs in the case of a degenerate charged fermion system such as the electron gas in metal [1]. Both effects can be easily understood, in a field-theoretic language, by calculating the one loop photon propagator in which the charged particle propagators are at finite temperature and density. (The two effects are in fact quite similar; for the degenerate fermion gas, the excitations generated by the propagating photon are particle-hole pairs which behave like plasma background). The non-Abelian analogues of these screening effects are of considerable interest especially in view of the possibility of obtaining hot and dense quark matter systems in heavy ion collisions [2]. A non-Abelian Thomas-Fermi effect can also be of interest in calculations of the equation of state for quark matter inside neutron stars. Non-Abelian Debye screening and related effects have been intensively investigated over the last few years [3-8]. It is well-known that a proper calculation of this effect involves summing up all the hard thermal loop Feynman diagrams [3]. This results in a gauge-invariant, nonlocal,effective mass term for the gauge bosons (or gluons in a chromodynamic context). This mass term has many nice properties being closely related to Chern-Simons and Wess-Zumino-Witten (WZW) theories [5,6].

In this letter we analyze the non-Abelian Thomas-Fermi screening for degenerate quark matter of finite baryon number. Since the quark contribution to the two-point function for gluons is similar to that for photons, there should be Thomas-Fermi screening for quark matter. For reasons of non-Abelian gauge invariance, as with Debye screening, there will be higher point contributions, the whole series again summing up to an effective mass term. This term will have the same structure as the effective action for hard thermal loops; the numerical value of the screening mass, however, will be determined by the chemical potential rather than temperature.

Consideration of the one loop two-point function shows that the screening mass $\sim g\mu$ where g is the coupling constant and μ is the chemical potential. It is then clear that a higher loop diagram in which such a term is inserted can give contributions of the same order for the integration range of loop momenta $\stackrel{<}{\sim} g\mu$. This is exactly as in the hard thermal loop case. One must therefore sum up diagrams with loop momenta $\sim \mu$ and external momenta $\stackrel{<}{\sim} g\mu$. This effective action must then be used for a self-consistent evaluation of the screeening mass. We obtain this effective action in what follows.

Let us start by considering the two-point function for gluons. The conserved baryon

charge is given in terms of the quark field $q(x)$ by $\int q^\dagger q$. With a chemical potential term $\mu \int q^\dagger q$ added to the action, the quark propagator is given by

$$S(x,y) = \langle Tq(x)\bar{q}(y)\rangle = \int \frac{d^3p}{(2\pi)^3} \frac{1}{2p^0} \big[\{\theta(x^0 - y^0)\alpha_p\gamma \cdot p\, e^{-ip(x-y)} + \bar{\beta}_p\gamma \cdot p'e^{ip'(x-y)}\} +$$
$$-\theta(y^0 - x^0)\{\beta_p\gamma \cdot p\, e^{-ip(x-y)} + \bar{\alpha}_p\gamma \cdot p'e^{ip'(x-y)}\}\big] \tag{1}$$

where $p^0 = |\vec{p}|$, $p = (p^0, \vec{p})$, $p' = (p^0, -\vec{p})$ and $\theta(x)$ is the step function. Also

$$\alpha_p = 1 - n_p, \qquad \beta_p = n_p \tag{2}$$

The distribution functions $n_p, \bar{n}_p$ corresponding to quarks and antiquarks respectively are given by

$$n_p = \frac{1}{e^{(p^0 - \mu)/T} + 1}, \qquad \bar{n}_p = \frac{1}{e^{(p^0 + \mu)/T} + 1} \tag{3}$$

The one-loop quark graphs are given by the effective action

$$\Gamma = -i\text{Tr }\log(1 + S\gamma \cdot A) \tag{4}$$

$A_\mu = -it^a g A_\mu^a$ is the gluon vector potential, t^a are hermitian matrices corresponding to the generators of the Lie algebra in the quark representation. In the above expression for Γ a functional trace is implied as well as the trace over the spin and color labels. The two-gluon term in Γ is given by

$$\Gamma^{(2)} = \frac{i}{2} \int d^4x\, d^4y\, \text{Tr}\big[\gamma \cdot A(x)S(x,y)\gamma \cdot A(y)S(y,x)\big] \tag{5}$$

Using equation (1) and carrying out the time-integrations we get

$$\Gamma^{(2)} = -\frac{1}{2} \int d\mu(k) \frac{d^3q}{(2\pi)^3} \frac{1}{2p^0} \frac{1}{2q^0} \Big[T(p,q)\big(\frac{\alpha_p\beta_q}{p^0 - q^0 - k^0 - i\epsilon} - \frac{\alpha_q\beta_p}{p^0 - q^0 - k^0 + i\epsilon}\big) +$$
$$T(p,q')\big(\frac{\alpha_p\bar{\alpha}_q}{p^0 + q^0 - k^0 - i\epsilon} - \frac{\beta_p\bar{\beta}_p}{p^0 + q^0 - k^0 + i\epsilon}\big) +$$
$$T(p',q)\big(\frac{\bar{\alpha}_p\alpha_q}{p^0 + q^0 + k^0 - i\epsilon} - \frac{\bar{\beta}_p\beta_q}{p^0 + q^0 + k^0 + i\epsilon}\big) +$$
$$T(p',q')\big(\frac{\bar{\alpha}_p\bar{\beta}_q}{p^0 - q^0 + k^0 - i\epsilon} - \frac{\bar{\beta}_p\bar{\alpha}_p}{p^0 - q^0 + k^0 + i\epsilon}\big)\Big] \tag{6}$$

where

$$A_\mu(x) = \int \frac{d^4k}{(2\pi)^4} e^{ikx} A_\mu(k)$$
$$d\mu(k) = (2\pi)^4\delta^{(4)}(k + k')\frac{d^4k}{(2\pi)^4}\frac{d^4k'}{(2\pi)^4} \tag{7}$$
$$T(p,q) = Tr\big[\gamma \cdot A(k)\ \gamma \cdot p\ \gamma \cdot A(k')\ \gamma \cdot q\big]$$

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and $\vec{p} = \vec{q} + \vec{k}$ in equation(6).

The $i\epsilon's$ can be taken to go to zero at this stage. They were introduced for convergence of time-integrations and contribute to the imaginary part. Here we are interested in screening effects which are described by the real part of the two-point function. (Also for many physical situations, the relevant imaginary part is that of the retarded function which is not directly given by the above time ordered function [6].) Further we are interested in a degenerate gas of quarks; it is therefore appropriate to consider $T \ll \mu$. As $T/\mu \to 0$, the antiquark occupation numbers $\bar{n}_p \to 0$ (for positive μ). Equation (6) then simplifies as

$$\Gamma^{(2)} = -\frac{1}{2} \int d\mu(k) \frac{d^3q}{(2\pi)^3} \frac{1}{2p^0} \frac{1}{2q^0} \Big[T(p,q) \frac{(n_q - n_p)}{p^0 - q^0 - k^0} + T(p,q') \frac{n_p}{p^0 + q^0 - k^0} \\ -T(p',q) \frac{n_q}{p^0 + q^0 + k^0} \Big] \tag{8}$$

As explained in the introduction, the relevant kinematic regime is $|\vec{p}|, |\vec{q}| \gg |\vec{k}|$, so that $p^0 - q^0 - k^0 \approx -k \cdot Q, p^0 + q^0 \pm k^0 \approx 2q^0, Q = (1, \vec{q}/q^0)$. Further,

$$T(p,q) \approx 8q^{0^2} tr(A_1 \cdot Q A_2 \cdot Q) \tag{9}$$
$$T(p',q) \approx T(p,q') \approx 4q^{0^2} tr(A_1 \cdot Q' A_2 \cdot Q + A_1 \cdot Q A_2 \cdot Q' - 2A_1 \cdot A_2)$$

where $A_1 = A(k)$, $A_2 = A(k')$, $Q' = (1, -\vec{q}/q^0)$. Expression (8) now simplifies as

$$\Gamma^{(2)} = -\frac{1}{2} \int d\mu(k) \frac{d^3q}{(2\pi)^3} tr \Big[\frac{dn}{dq^0} \frac{A_1 \cdot Q A_2 \cdot Q}{k \cdot Q} 2\vec{Q} \cdot \vec{k} - \frac{n}{q^0} (A_1 \cdot Q' A_2 \cdot Q + \\ A_1 \cdot Q A_2 \cdot Q' - 2A_1 \cdot A_2) \Big] \tag{10}$$

Using $\int d^3q \frac{dn}{dq^0} f(Q) = -\int d^3q \frac{2n}{q^0} f(Q)$, and some properties of $\vec{Q}$-integration, $\Gamma^{(2)}$ simplifies to

$$\Gamma^{(2)} = -\frac{1}{2} \int d\mu(k) \int \frac{d^3q}{(2\pi)^3} \frac{n}{2q^0} 8 \, tr \Big[2A_{1+}A_{2-} - \frac{k \cdot Q'}{k \cdot Q} A_{1+}A_{2+} - \frac{k \cdot Q}{k \cdot Q'} A_{1-}A_{2-} \Big]$$
$$= -\frac{1}{2} \int d\mu(k) \Big(\frac{\mu^2}{4\pi^3}\Big) \int d\Omega \, tr \Big[2A_{1+}A_{2-} - \frac{k \cdot Q'}{k \cdot Q} A_{1+}A_{2+} - \frac{k \cdot Q}{k \cdot Q'} A_{1-}A_{2-} \Big] \tag{11}$$

where $A_+ = \frac{A \cdot Q}{2}, A_- = \frac{A \cdot Q'}{2}$ and we have taken the limit of $T \to 0$ (small compared to μ). As with hard thermal loops, we can transform this back to coordinate space and write

$$\Gamma^{(2)} = -\frac{\mu^2}{4\pi^3} \int d\Omega \, K^{(2)}(A_+, A_-) \tag{12}$$

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where

$$K(A_+, A_-) = \Big[\int d^4x \; \mathrm{tr}(A_+ A_-) + i\pi I(A_+) + i\pi \tilde{I}(A_-)\Big]$$

$$I(A_+) = i \sum_2^\infty \frac{(-1)^n}{n} \int d^2 x^T \frac{d^2 z_1}{\pi} \cdots \frac{d^2 z_n}{\pi} \frac{tr[A_+(x_1)\cdots A_+(x_n)]}{(\bar{z}_1 - \bar{z}_2)\cdots(\bar{z}_n - \bar{z}_1)} \tag{13}$$

$\int d\Omega$ defines integration over the orientations of $\vec{Q}$. $K^{(2)}$ in equation (12) denotes the terms in K which are quadratic in $A's$; z and $\bar{z}$ are the Wick-rotated versions of $x \cdot Q'$ and $x \cdot Q$ respectively and x^T is transverse to $\vec{Q}$, i.e $\vec{Q} \cdot \vec{x}^T = 0$. $\tilde{I}(A_-)$ is obtained from $I(A_+)$ by $z \leftrightarrow \bar{z}$. $I(A_+)$, as explained elsewhere, is essentially the eikonal function for Chern-Simons theory. $K(A_+, A_-)$ can be related to the WZW-action for a hermitian matrix $M^\dagger M$ defined in terms of $A_\pm$.

Consider now the three-point function. One has twenty-four terms with denominators involving different combinations of loop momenta and external momenta. The simplification of this expression proceeds in much the same way as for $\Gamma^{(2)}$ - one can neglect terms with denominators that are of the order of μ and leave the terms that involve differences between loop momenta, which are of the order of $k \ll \mu$. After that, the relevant contributions will be

$$\Gamma^{(3)} = \int \frac{d^3 q}{(2\pi)^3} \, 2 \Big\{ \frac{n_p}{k_2^0 + p^0 - q^0} \frac{1}{r^0 - q^0 - k_3^0} + \frac{n_r}{q^0 - r^0 + k_3^0} \frac{1}{p^0 - r^0 - k_1^0} + \frac{n_p}{p^0 - r^0 - k_1^0} \frac{1}{p^0 - q^0 + k_2^0} \Big\} \, \mathrm{Tr}(A_1 \cdot Q A_2 \cdot Q A_3 \cdot Q) \tag{14}$$

where p, q, r are the loop momenta, k_1, k_2 and k_3 are the momenta of external gluons and $\vec{p} = \vec{q} - \vec{k}_2, \vec{r} = \vec{q} + \vec{k}_3$. After performing the dq^0-integral the final expression is

$$\Gamma^{(3)} = \frac{\mu^2}{(2\pi)^3} \int d\Omega \Big(\frac{k_2 \cdot Q' - k_2 \cdot Q}{k_1 \cdot Q k_2 \cdot Q} + \frac{k_3 \cdot Q - k_3 \cdot Q'}{k_3 \cdot Q k_1 \cdot Q} \Big) \, \mathrm{Tr}(A_1 \cdot Q A_2 \cdot Q A_3 \cdot Q) \tag{15}$$

Using definition (13) one can show that

$$\Gamma^{(3)} = -\frac{\mu^2}{4\pi^3} \int d\Omega \; K^{(3)}(A_+, A_-) \tag{16}$$

The fact that the same coefficient appears in both (12) and (14) is crucial; this guarantees gauge invariance of the full effective action Γ. The non-Abelian gauge-invariant completion of $K^{(2)} + K^{(3)}$ is given by the full K of equation (13). The final answer is thus

$$\Gamma = -\frac{\mu^2}{4\pi^3} \int d\Omega \; K(A_+, A_-) \tag{17}$$

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We now turn to the question of the coefficient of K in equation (17). In calculating the screening effects using Γ, we should not change the Lagrangian for the theory, which may be, say, chromodynamics; we should only rearrange terms in the perturbative expansion and sum certain classes of diagrams. There are then two different but closely related ways of proceeding. We write the action as

$$
\begin{aligned}
S &= S_0 + \Delta \int d\Omega \ K \\
S_0 &= S_{QCD} - m^2 \int d\Omega \ K
\end{aligned}
\tag{18}
$$

S_0 is used to define propagators and vertices and Δ is treated as being nominally one loop order higher than S_0. In the first case, we use the value as calculated above (or the analogous value for hard thermal loops) for m^2. After calculating higher order corrections, Δ is set to m^2 (so that $S \to S_{QCD}$). The corrections are in general non-vanishing and this is useful if the corrections are small compared to the lowest order value. The alternative is to keep m^2 as an arbitrary parameter and choose Δ (as a function of m, say $\Delta(m)$) so as to cancel out all the corrections. Upon setting Δ to m^2, we get a gap equation, $\Delta(m) = m^2$, which can be solved for m. In the case of the hard thermal loops, the first approach is satisfactory. The relevant distribution for the momentum-integration is

$$
f(q)dq = \frac{q^2 dq}{e^{q/T} + 1}
\tag{19}
$$

The lowest order calculation only evaluates the contribution from the region of $q \gtrsim T$. The probability contained in the region $q \geq \frac{1}{2}T$ for the distribution is approximately 0.99. We can therefore expect that the neglect of the low q-regime is not very significant for the numerical value of the screening mass (within a calculational scheme with resummations as explained above). For the case of Thomas-Fermi screening, the relevant distribution is

$$
f(q)dq \approx \theta(\mu - q)q^2 dq
\tag{20}
$$

The probability contained in the region $q \geq \frac{1}{2}\mu$ is now 0.875. We thus expect that the lowest order value of the coefficient of $-\int d\Omega \ K$ in equation (15), viz., $\mu^2/4\pi^3$, is somewhat less accurate than the analogous quantity for hard thermal loops. In this case a self-consistent calculational scheme for including higher order effects might be more appropriate.

After this work was completed, we became aware of a paper by Cristina Manuel where non-Abelian Thomas-Fermi screening is discussed in a kinetic theory framework and results

similar to ours are obtained [9]. Our approach, based on evaluation of Feynman diagrams, is complementary to this work. We thank Cristina Manuel for bringing this work to our attention.

References

1. See, for example, A.L.Fetter and J.D.Walecka, "Quantum theory of many particle systems", McGraw-Hill, 1971.

2. See, for example, C.P.Singh, *Phys.Rep.* **236** (1993) 147.

3. R.Pisarski, *Physica A* **158** (1989) 246; *Phys.Rev.Lett.* **63** (1989) 1129; E.Braaten and R.Pisarski, *Phys.Rev.* **D42** (1990) 2156; *Nucl.Phys.* **B337** (1990) 569; *ibid.* **B339** (1990) 310; *Phys.Rev.* **D45** (1992) 1827.

4. J.Frenkel and J.C.Taylor, *Nucl.Phys.* **B334** (1990) 199; J.C.Taylor and S.M.H.Wong, *Nucl.Phys.* **B346** (1990) 115.

5. R.Efraty and V.P.Nair, *Phys.Rev.Lett.* **68** (1992) 2891; *Phys.Rev.* **D47** (1993) 5601.

6. R.Jackiw and V.P.Nair, *Phys.Rev.* **D48** (1993) 4991.

7. J.P.Blaizot and E.Iancu, *Phys.Rev.Lett.* **70** (1993) 3376; *Nucl.Phys.* **B 417** (1994) 608.

8. R.Jackiw, Q.Liu and C.Lucchesi, *Phys.Rev.* **D49** (1994) 6787; P.F.Kelly *et al, Phys. Rev. Lett.* **72** (1994) 3461; *Phys.Rev.* **D50** (1994) 4209; V.P.Nair, *Phys.Rev.* **D48** (1993) 3432; *ibid.* **D50** (1994) 4201; A.K.Rebhan, *Phys.Rev.* **D** (1993) 3967, *Nucl.Phys.* **B 430** (1994) 319.

9. C. Manuel, *Phys.Rev.* **D53** (1996) 5866.